MATHEMATICS TEACHING

THE BULLETIN OF THE
ASSOCIATION for TEACHING AIDS in MATHEMATICS

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No. 9 — APRIL 1959 3/6

MATHEMATICS TEACHING

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BELMONT PRESS LTD., Queenathorpe Road, Sydenham, S.E.26

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No. 9 APRIL, 1959 3/6

This Bulletin appears three times yearly.

EDITORIAL

TEACHING MATHEMATICS IN SECONDARY SCHOOLS

Ministry of Education Pamphlet No. 36. H.M.S.O. 1958. 6/-

At long intervals the Ministry of Education publishes pamphlets on teaching methods. These pamphlets must do two things: they must report to the nation on the state of mathematics in the schools as H.M. Inspectors see them, and make suggestions for the improvement of its teaching. One therefore gains the impression that although there is some sound sense in this pamphlet the writers had an eye to the fact that it would be read and quoted by posterity. It appears to be written for those who think about, talk about and try to influence the man on the job rather than for the teacher as an active member of a classroom community.

For this reason the pamphlet seems to lack detail, detail which such people are able to provide for themselves, and background which they already possess.

The book consists of an introductory section on the development of the teaching of mathematics, a brief section on the primary stage, a fairly full if rather general section on the syllabus for the grammar school including the sixth form, followed by a brief consideration of the ordinary pupil's needs. Then follows a chapter on the correlation with other subjects, a chapter on classroom practices and organisations. The pamphlet ends with a full chapter on the History of Mathematics.

There are many threads running through it: the marvellous development of mathematics teaching in the last fifty years, suggestions for syllabuses, comfort for exponents of "practical" methods. It requires very careful reading and re-reading before all its parts may be adequately integrated, before all its stimuli have fully been appreciated. The book has mainly been written for the Grammar School: that much is evident and its echoes have been heard in the Secondary Modern Press. But perhaps this implies the secondary modern curriculum will contain the same mathematics as any other school; perhaps the "modern" part of the title has not been justified; perhaps the secondary modern schools have not *in the main* produced anything so significant as the mathematics curriculum of the secondary grammar school.

The report like its predecessors is imbued with the principles that the more mathematics a teacher knows, the better is his equipment as a teacher of the subject, a principle which meets with surprisingly vehement opposition.

There are many research topics indicated for a teacher. For example:

(i) A detailed analysis of what the S. M. School has accomplished.

(ii) Would a truer understanding of geometry be fostered if the tools were varied: (paperfolding, dividers and ruler, ruler-and-fixed-circle, ruler-only construction)?

(iii) Should we teach wave motion to the mathematical sixth?

(iv) Develop work units on a card system.

(v) Work out a course in geometrical design for girls.

(vi) Find useful interesting material for girls' maths lessons in Needlework, Housecraft, Art and Architecture.

(vii) Develop geometry on Clairaut's pattern.

Perhaps the novel emphasis is the insistence on the teaching of mechanics at all levels in the secondary school. The most irritating feature is the way in which broad vistas are opened and then shrugged off.

The book does argue that a successful approach starts from the children and their interests; the authors firmly believe that concrete (physical) experience should form the basis from which mathematics can be extracted. Again and again it emphasises the uses of discussion, the development and application of a principle, the use of language to ensure a thorough understanding of the mathematics being developed. The various phases of classroom activity are put clearly into perspective and many teachers would do well to note the section on marking and written work. There are many hints to the writers of textbooks and people concerned with the development of syllabuses as to topics to be included and as to desirable correlations between subjects, but the man on the spot will find few lamps to lighten his darkness.

A Headteacher will find it difficult to use as a yardstick with which to measure the success of his mathematics department. The ordinary product of the training colleges will find much that is incomprehensible. The revolutionary will bemoan the absence of a clarion call, its rather subdued and conservative call to change. The diehard pen-and-ink traditionalist will fulminate against the absence of tips and wrinkles and the "encouragement of the mathematical mystique. Let's get back to sums". In fact the book is decidedly left of centre, but it is forward looking. There is much food for thought and many points to reflect upon: not least the Minister of Education's introduction:

"While the precise content of a mathematical education is a matter of opinion, we can be certain of one thing, that mathematics must be taught as a language. Power of expression in this language, and its ready application, cannot be cultivated too early; *the techniques of the subject can await time and opportunity.*"

The italics are mine, the punctuation is the politicians! C.H.

KEEPING UP

A University mathematician often spends some time regularly trying to "keep up with the literature"—that is to keep up with recently published research in those parts of mathematics which interest him.

What does this entail?

A cursory glance at *Mathematical Reviews* gives some idea. These reviews are published eleven times a year by the American Mathematical Society. They are sponsored by thirteen other mathematical societies from all parts of the world, and they are printed in Holland with contributions in English, French, German and Italian. Each orange-covered issue contains over one hundred pages, and the aim is merely to list and summarise fresh mathematical papers as they appear. Generally speaking the papers listed all contain mathematics that is reckoned to be new, as the reviews are little concerned with textbooks or with articles which give popular expositions of existing knowledge.

A typical issue (February, 1958) shows the speed at which mathematics is

advancing. This issue contains 126 pages and lists 712 articles classified under the headings, Foundations, Theory of Sets, Logic; Algebra; Theory of Numbers; Analysis; Topological Algebraic Structures; Topology; Geometry; Numerical Analysis; Probability; Statistics; Physical Applications; Other Applications; History, Biography; Miscellaneous. 414 of the papers listed are in English, 98 in Russian, 77 in French, 72 in German, 18 in Italian, 7 in Ukrainian, 6 in Czech, 5 in Roumanian, 2 in Portugese, 2 each in Polish, Dutch, Hungarian and Spanish, and one each in Estonian, Bulgarian, Serbo-Croatian and Chinese. (These counts are unchecked and are liable to be out by a few either way.) Naturally the distribution varies from month to month, but this analysis probably gives a reasonable indication of the languages which matter most to Western mathematicians today. Is this an argument for teaching Russian in schools?

It is difficult to say how widely these papers are understood; but few men could claim close familiarity with the ideas discussed in more than a fraction of them. Outside their own particular field experts understand little more than you or me.

A return visit to the library of my student days shows banks of shelves devoted to branches of the subject which have developed since I left, and the pages of *Mathematical Reviews* show us more than 7,000 new papers each year. The prospect is frightening, but if we speak admiringly of "progress" then this is what we admire.

Now what can the ordinary mathematics teacher do about it all? Is the task of keeping up quite hopeless? Must we hobble through lessons and lectures teaching only the mathematics of the past? We print this journal in the belief that we need not. Every teacher can aim to learn at least some small amount about the fresh discoveries being made. This is not to suggest that teachers can hope to keep pace with the whole of the subject, or even with a more restricted field, but they can be determined that their own knowledge shall not stand still. The teacher must continue to learn mathematics himself, keeping his mind active if only in some small part of the subject.

Puzzles with numbers often intrigue people, even if they have no claim to be mathematicians at all. The results of some modern work in the Theory of Numbers can be understood by schoolboys even if the proofs are beyond them. Must we prove everything we do? Topics can have news value even in a mathematics lesson, and it is the business of a school to reach the frontiers of knowledge. Schools have made differential analysers with Meccano. Is a transistor computer beyond them?

"Applied Mathematics" today means far more than the school syllabus of mechanics. There are the new sciences of the applications of mathematics to human behaviour, and to business management and the allocation of material resources. Why not use these ideas in the classroom to discuss examination strategy and school organisation?

The present situation is not basically new. Long division was once regarded as a difficult subject, and many universities did not teach it because they thought it too hard. After the Renaissance a man had to learn vastly more in order to become educated—but men managed. They can manage again today if only we believe it possible.

THE SPATIAL CONCEPTS OF THE CHILD

P. C. DODWELL

In a previous article in this journal (November, 1957) I showed how the number concepts of the child evolve through the first seven years of life, and how this was demonstrated in the ingenious but simple investigations of Jean Piaget. One of Piaget's most important contributions was to elaborate a method of studying the use of number concepts by children which brings out not only the conceptual limitations of the young child, but also points to the factors which govern those conceptual limitations. One of the most important of these factors—perhaps the most important—is what might be called the hegemony of perception; the average child of, say, five years is almost completely dominated in its thinking about quantities, serial order and number by what it perceives at any given moment (cf. Piaget, *The Child's Conception of Number*, or op. cit.). It would therefore seem that the child's conception of geometry and its ideas concerning space should constitute an interesting field of study, since the relationship between what is perceived and the conceptual framework used is especially intimate in geometry, at least in the elementary treatment of the subject.

Piaget's investigations in this field are described in *The Child's Conception of Space*, by J. Piaget and B. Inhelder (Routledge & Kegan Paul, 1957) and they constitute a remarkable contribution to our knowledge of the way the child thinks about, and represents to himself, the space around him. As in the earlier work on number, Piaget is concerned to show, through quite simple demonstrations and tests, what geometrical notions the child can grasp and use in solving problems, answering questions, identifying objects, drawing what it sees, and so on. Also, as in the work on number, he ignores the findings and traditions of geometry teaching in schools, and looks for the sources of those abilities of the child which *underlie* the capacity to learn and understand the geometry it is taught in school, and he looks for those sources in the child's everyday commerce with the normal environment in which it lives, in the sorts of problem involving spatial representation or manipulation which the child normally meets and tries to solve with whatever spatial concepts it may have. (Whether these spatial concepts are innate or learned, whether they can be influenced by training or not, are further questions, as yet undecided).

I think it is widely accepted amongst teachers of geometry that the sort of geometry it is important for children to learn is of the Euclidean variety. The fact is perhaps not widely enough recognised that this geometry, as developed historically and as usually taught, occupies a rather special position in the field of mathematical and geometrical thinking. It is of course a geometry which has immediate practical application in the physical world, and no doubt this is one of the reasons for its development comparatively early in the history of mathematics; also this is one of the reasons for its introduction as the first, basic, geometry in the school curriculum. The modern trend towards instruction with the aid of concrete demonstration, models and so on, is peculiarly suited to the characteristics of Euclidean geometry, just because it *is* the geometry which "fits" the world of physical objects and events—a geometry whose interpretation and application is so immediate and obvious that it is only in relatively recent times that the importance of distinguish-

ing between the geometry as a formal system and its intepretation in physical terms has been fully recognised.

It should be realised, however—as readers of my earlier article will no doubt have grasped by now—that the ability to understand this system of geometry should not be looked at as something "given" in the intellectual development of the child; in Piaget's terminology, one can expect that this ability will depend on the prior acquisition of a system of logical operations related to the notions of physical quantity, extension, proportionality and measure. Essentially what Piaget is interested in is the process of development of such systems of operations (called "groupements" in French, and usually translated as "groupings") which are prerequisites of the ability to understand Euclidean geometry and to reason "operationally" about space. It would be a mistake to think that, because Euclidean geometry is somehow thought of as "basic", the concepts, operations or logical rules which govern its use are primitive notions which in some way appear in the child at a certain age fully armed, as it were, like Athene springing from the forehead of Zeus. Recalling Piaget's work on number, we can expect to find a long process of development of spatial concepts and ideas up to the stage of concrete operational thinking about space, and it is the investigation of this process with which I shall mainly be concerned here.

Put another way, one could say that we should not expect, just because Euclidean geometry fits the space of physical objects and events in which we live, that the spatial concepts implicit in Euclidean geometry are the first spatial concepts a child can grasp, understand and use. This idea springs from the notion, which most of us hold implicity, that Euclidean geometry is the primitive, elementary basis on which other systems are founded by generalisation of the relations and properties of that geometry.

Logicians and philosophers like Frege, Russell and Whitehead demonstrated at the formal level that the concept of number is logically complex and can be "reduced" to simpler concepts concerning order, class and correspondence, and Piaget showed how this is reflected in the development of the concept of number in the child from simpler activities and operations concerned with ordering, classifying and constructing correspondences between sets of objects. So too in the case of geometry the demonstration that the "basic" Euclidean geometry is in fact logically complex, and that more primitive concepts like "closed surface", "neighbourhood", "inside" and "outside" yield a more fundamental notion of the attributes of space (topology) is reflected in Piaget's demonstration of how these later concepts have to be developed by the child before it can even begin to grasp the ideas of Euclidean geometry. Moreover, the attainment of such apparently "obvious" concepts as that of "straight line"—which is even perceptually obvious —the composition of a line from a set of points, the number of sides and angles in a rectilinear figure, are shown by Piaget to involve an active process of construction and experiment on the part of the child.

One might ask why, if these activities are universal and a necessary antecedent to the development of concepts of number and space—and indeed other concepts germane to other domains of knowledge—no one has noticed the fact before. The answer, I suppose, must be that no one has thought very carefully about the genesis of factual knowledge in this way before. On the one hand philosophers, worried

by problems of epistemology, have been content to analyse concepts about space, time and so on, without bothering to find out whether the actual genesis of these concepts in young children could throw any light on their philosophical arguments; on the other hand teachers have, I think, tended to accept too easily traditional ideas about the way teaching should be promulgated, or have assimilated too freely ideas fashionable in some current psychological school* without being too careful to observe whether the child's actual behaviour and development can be understood with the aid of such ideas. I am not suggesting that Piaget's ideas should be taken over uncritically by mathematics and geometry teachers—but I do suggest that they may yield a valuable insight into the basic process of development through which the child comes to be able to understand the concepts of number and space, and hence suggest ways and means of introducing the subject of geometry in the most meaningful way for the child, i.e. in a way which meets half-way its already partly developed spatial concepts. This would be of use if it were found that rearrangement of a teaching syllabus taking into account Piaget's findings did in fact aid children in the understanding of the relevant subject matter. So far as I know this has as yet not been undertaken.

To turn now to Piaget's experiments:

It is well known that very young children find it difficult to discriminate between simple geometrical shapes such as squares, triangles and crosses; Piaget investigated in a fairly systematic manner the way in which perception of shapes develops, but he was interested not so much in the simple visual discrimination of one shape from another, as the *perceptual activity* which constitutes an important factor in coming to understand the differences between shapes. It is quite conceivable that a young child could be taught to respond differentially to two shapes (e.g. by a process of conditioning) without really having a clear idea of what the difference between them is; a child may be able to name a square and a circle correctly, and always choose the correct one of the two when asked, but it does not follow that he has an understanding of the properties which differentiate the shapes. If one asks such a child (about three years old) to *draw* a square and a circle, one finds that it draws two practically identical shapes which can best be described as closed curves with more-or-less smooth outlines! Regular curves, straight lines and angles—let alone approximations to right angles—seem to mean nothing to the child, and the reason for this, Piaget argues, is that the child is only able to "represent" the shapes to itself topologically. Topologically a square and circle cannot be discriminated from each other, since both are simply closed surfaces, hence the child draws both of them similarly. The same is true of other shapes, which, as Euclidean shapes, are very different from one another. It could be argued that the failure of the child to draw the square and circle "properly" is due to a simple inability to draw correctly, and has nothing to do with the intrinsic ability of the child to "represent" shapes to himself. By an ingenious experimental device, Piaget showed that this argument cannot be sustained. He arranged a test situation in which the child could not see the shape to be identified, but could only feel it (this has sometimes been called "haptic" perception); the

* An example of this is the uncritical taking over of certain of the ideas of Gestalt psychology into the teaching of reading.

child is seated at a table in front of a small screen, places its arms behind the screen and is given the shape (cut from cardboard or plywood) to hold. It is then asked *either* to draw the shape *or* to pick it out from a selection of shapes displayed on a tray. It was found that the ability to draw reasonable adequately lags a little behind the ability to pick out the correct shape, but not very much; therefore the inability to "represent" the shape cannot be attributed simply to lack of drawing skill. It might be objected that this method of investigation presents the young child with a highly artificial situation, and that it is not surprising that it finds difficulty in identifying shapes by tactile-kinaesthetic cues alone. Against this it can be pointed out (a) that very small children, in the first few months of life before they gain control of the upper part of the trunk, have considerable practice at identifying things (including parts of the body) not within the visual field in this way, and (b) that they have little difficulty at about three years old in identifying common objects such as a key or a spoon without the aid of vision.

As in most of his other investigations, Piaget found that there is a regular series of fairly well-defined stages through which the child's ability to identify shapes by this means progresses. In the earliest phase (starting at *approximately* three-and-a-half years) the child is able to identify shapes only in terms of topological attributes; thus whilst a square and circle are indistinguishable from each other or any other closed surface, they can be distinguished readily from shapes with different topological characteristics, e.g. shapes such as those shown in figure 1.

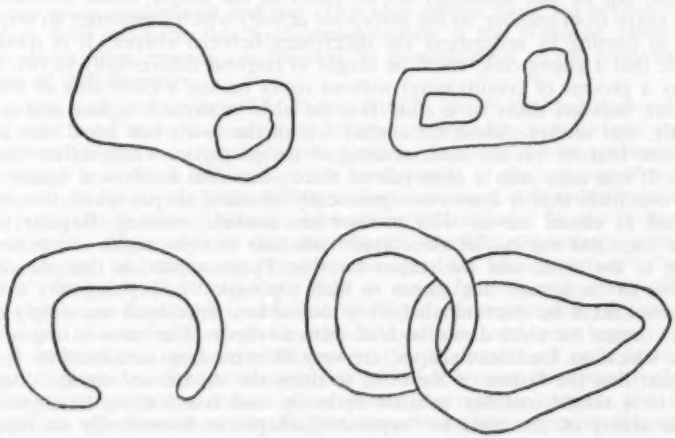

Fig. 1 Shapes with differing topological characteristics.

These shapes are also usually distinguishable from one another. The second stage, lasting from about four-and-a-half to six or seven years is divided into three sub-stages. In sub-stage A a crude differentiation between rectilinear and curvilinear shapes appears, but different rectilinear shapes (e.g. square and triangle) or different curvilinear shapes (e.g. circle, ellipse) are not differentiated among themselves. There is as yet no systematic exploration of the shapes, but chance encounters with relevant features (e.g. angles) are utilised in making identifications; in the first stage the shape was simply grasped between the palms of the hands with hardly any exploration. In the second sub-stage (B) there is a progressive differentiation of shapes according to their angular properties and dimensions (e.g. length of side) and the active search for significant cues enters the picture, although this is still not systematic. The third sub-stage (C) is characterised by the successful differentiation of rhombus and trapezium, and the recognition of more complex shapes such as stars and crosses, but still with some errors. The third stage shows methodical exploration of shapes, counting of angles, estimating relative lengths, etc. Complex forms are readily identified and correctly drawn. This stage, one might say, already shows the characteristics of "operational" activity.

This development from recognition in terms of topological qualities to recognition of Euclidean shapes is also shown in a study of children's drawings. Piaget collected children's drawing of 21 shapes (models), varying from shapes showing simple topological relationships to relatively complex geometrical forms. A selection of these is shown in figure 2. Figure 3 shows some typical results of attempts to reproduce these figures at the first stage. It will be noticed that, whereas the topological relationships are correctly reproduced, representation in Euclidean terms is not found, although attempts to represent features such as angles do occur: the important thing is that these attempts are not at all *co-ordinated with* the main topological feature of the shape, the fact that it is a simple closed surface.

We all know, of course, that young children are not good at copying shapes; the important thing Piaget has demonstrated is the *pattern* which is discernible in the development of spatial representation, a pattern which makes the child's mistakes appear not just as random or meaningless errors, but as expressions of the conceptual relationships it is able to grasp at any particular stage. There are other experiments demonstrating the importance of topological relationships in the spatial conceptions of the young child—concerned with linear and circular order, the study of knots, continuity and the composition of a line from points, etc.—but the list is too long for their description here.

It will be remembered that the first step forward from representing shapes in purely topological terms was found to be the differentiating of rectilinear from curvilinear shapes (sub-stage II B), without regard for metric properties (length of side, size of angle etc.). These are characteristics which are preserved under projective transformation, which already gives us a hint about the next of Piaget's discoveries: that is, that the child learns to understand projective properties before it can grasp Euclidean relationships, and the ability to operate with these latter is dependent on learning to co-ordinate different perspectives (which, from the practical point of view, means different "viewpoints") one with another. Of great interest here is the manner in which the child represents, or constructs, one of the principal projective elements, the straight line. As Piaget says: " . . . nothing is

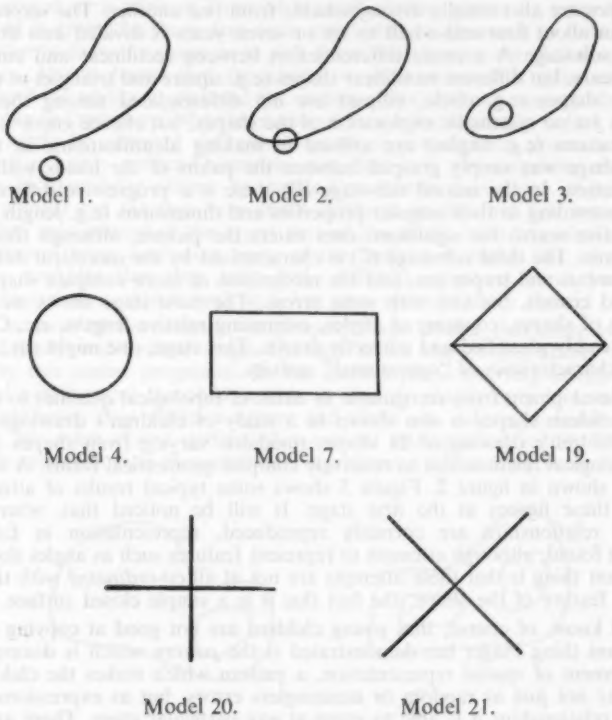

Fig. 2. Examples of models.

more suited to illustrating the vast difference between perceptual and representational space than to see how backward in imagining or constructing a straight line are children who have long been able to recognise straight lines perceptually" (*The Child's Conception of Space*, p. 156).

The materials used for these experiments are a table (square, rectangular or round) and a set of "telegraph poles" made of matchsticks on plasticine bases. The child is told that a straight line of poles runs along beside a perfectly straight road: the first and last poles are put in position and the child is asked to set up the rest. Three main stages are again discernible, corresponding with those found in the previously described experiments. In the first, the child simply sets down the poles in proximity to one another, in a line which is by no means straight, but is somewhere between the two end poles; again, the topological character of

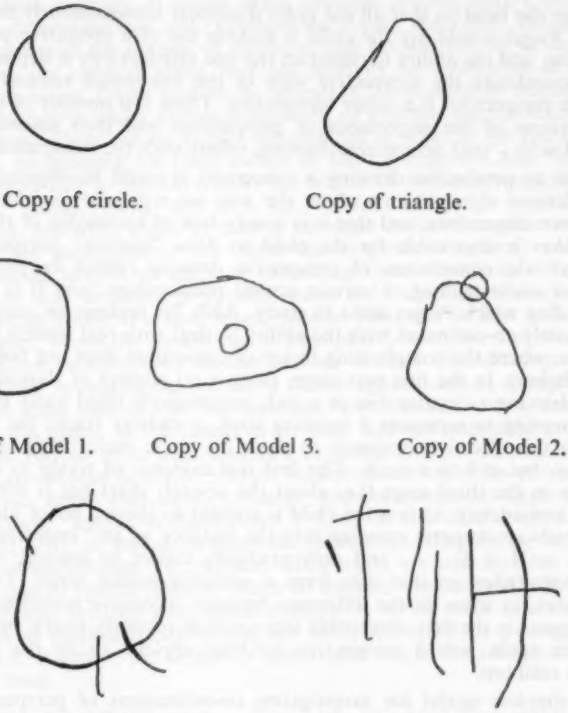

Fig. 3. Sample drawings, Stage I.

proximity of points which make up a line is preserved but almost nothing else. Stage II is especially interesting: if the end-poles are placed at two adjacent corners of the square (or rectangular) table, the childs tends to place the remaining poles in a fairly straight line between them, and also more evenly spaced out than before. However, it can easily be shown that the child has simply used the straight edge of the table as a "model" straight line, for if the end poles are moved, for instance to the mid-points of adjacent sides, it is no longer able to construct an adequate straight line. When the experiment is done with the round table, and the end-poles placed at two points on the circumference, Piaget found that the child tends to make his line follow the curved edge of the table! Towards the end of this stage the table contour plays less and less part in determining the positions of the poles. The third stage is characterised by ability to construct a completely "independent" straight line, and this is nearly always accompanied by "sighting",

i.e. placing the head so that all the poles disappear simultaneously behind the near end-pole. Piaget would say the child is making use of a projective property of the straight line, and the ability to construct the line satisfactorily is dependent on being able to co-ordinate the perspective view of the line which reduces it to a point with other perspectives (i.e. other viewpoints). There is a number of other tests and demonstrations of the importance of perspectives and their co-ordination, some concerned with actual perspective drawing, others with the manipulation of models.

As far as perspective drawing is concerned, it could be objected that there is a large element of convention about the way we represent the three-dimensional world in two dimensions, and that it is simply lack of knowledge of the conventions which makes it impossible for the child to draw "correct" perspective. On the other hand, the conventions of perspective drawing cannot be properly applied without an understanding of certain spatial relationships, and it is precisely this understanding which Piaget seeks to study. Also, his findings on perspective drawing are closely co-ordinated with the ability to deal with real models from different viewpoints, where the complicating factor of convention does not hold in the same way (see below). In the first two stages there is no attempt at showing perspective, either in drawing a circular disc or a rod, progressively tilted away from the child, or in attempting to represent a receding road or railway track; the first primitive attempt in this direction is simply to draw the circle increasingly smaller as it is tilted away, but still as a circle. The first real evidence of trying to show perspective comes in the third stage (i.e. about the seventh year) but is still rudimentary and lacks consistency; at first the child is content to show a set of identical objects (such as railway sleepers) receding into the distance as an "intensive"* series, i.e. simply $A > B > C \ldots$ and only gradually comes to attempt to relate their relative magnitudes, so that they form a smoothly-graded series. This co-ordination of series, in terms of the difference between successive members of the series Piaget suggests is the first observable step towards properly metric operations. One may notice again how a perspective attribute appears to be the forerunner of Euclidean relations.

The simplest model for investigating co-ordinations of perspectives consists of a geographical model of three mountains, each with distinctive features of size and shape, and having on its summit a house, cross, etc. The child is shown the model from one viewpoint and asked to draw it, or to pick out from a set of pictures the one which shows the mountains "from this point view". This is not too difficult, but the child in the earlier stages has great difficulty in deciding what the model will look like from any other point of view than the one it at present occupies. Again there is a fairly regular progression in the ability to emancipate itself from its immediate viewpoint and co-ordinate various perspective possibilities with each other.

I hope this brief sketch of some of the investigations Piaget has undertaken with younger children gives some idea of the importance of "pre-Euclidean" concepts in the development of their ideas about space, and their capacity to

* Piaget talks of "intensive" and "extensive" quantities. "Intensive" quantities are orderable for magnitude, but not simply additive (e.g. temperature, pain); "extensive" quantities are additive and hence must be measurable (e.g. length, volume).

imagine and think spatially. It would make tiresome reading were I to catalogue all the other demonstrations and experiments Piaget describes, but one other group, concerned with the notions of similarity and proportions, deserves mention. Needless to say the child of about four has virtually no conception of similarity in the Euclidean sense; Piaget studied the genesis of the concept by having children copy cardboard figures under various conditions, or estimate the similarity between perceived shapes. There are no results for children at stage I, for the concept of similarity has no meaning for them except in the topological sense. At Stage II (about four-and-a-half to six-and-a-half) the notion of similarity starts to appear, but only in a "global" form: that is to say, children will judge squares different from rectangles, but when asked to draw a rectangle similar to a given model tend to exaggerate the rectangularity (i.e. make the long sides too long) and have no idea of comparing lengths of sides, or measuring them. Similarity of triangles is first thought of only in terms of parallel sides, and "equality of angles" is only seen as a relevant factor at a later stage; parallel sides can at first only be drawn *round* the model, where the parallelism is perceptually obvious, and the similarity of two shapes not superimposed on one another is more difficult to grasp. Piaget demonstrated that in judging the similarity of two figures such as triangles, children in this second stage tend to take into account only *one* factor at a time, such as the height of the figure, or the length of its base, and are only able to make real progress when two or more attributes can be considered simultaneously (cf. the judgments of amounts of liquid in different beakers described in my previous article; there also it was found that ability to judge volumes adequately depended on the ability to perform "logical multiplications"). Only at the third stage do attempts to consider equality of angles, proportions of corresponding sides, and to make rudimentary measurements, etc., appear. Here again we can say that the child's conceptualisation of spatial relations starts to become "operational". Relationships are organised and ordered into stable systems in which each operation has an inverse, relations become transitive and not easily disrupted by perceptual factors as before. Still, however, the ability to understand Euclidean relationships is at a fairly simple level.

How relevant is all this to the teaching of geometry? Two points may have struck the reader already: first, this work is perhaps not very relevant to the teaching of geometry, since the subject is usually introduced after the onset of Stage III, and secondly it would be necessary to know how far Piaget's results are general before making them an important consideration in deciding how best to teach geometry and when to introduce the subject. However, it seems very plausible to suppose, *vis-à-vis* the first point, that if one introduces a Euclidean system to the child straight away this will be less easily assimilated (for the child's Euclidean concepts are still rather crude) than if one starts with the more primitive and better understood pre-Euclidean ideas and shows how Euclidean geometry grows out of these as a progressive elaboration of spatial concepts; it may well be that the easiest way for the child to grasp the invariance of relative position, of magnitudes and measurements, is to grasp them as extensions and elaborations from topological properties, and through the process of progressive co-ordination of perspectives (viewpoints). This is only a suggestion, of course, and would require experimental verification. As for the second point, it is true that we do not know

how general Piaget's findings may be; he seldom mentions the numbers of children used in his investigations, nor how variable results are from child to child, but it does seem that his reported results are sufficiently interesting to make further experimentation worth while and even important, for their educational implications —if validated—could be considerable. Such further experimentation should not be beyond the capabilities of a teacher sufficiently interested in the teaching of geometry, and if this article stimulates readers to probe further and with a new orientation into the spatial concepts of children it will, I think, have served a useful purpose.

THE SATISFACTION OF MATHEMATICS

E. M. GREGSON

We all share in appreciating the beauty and wonder of the universe and man's understanding of its secrets. It is our inheritance; and it is the privilege and the duty of adults to help children to grow into seeing the wonder, the rhythm and the pattern of their world. Indeed for Plato this was the aim of education. We may wonder at the behaviour of iron filings scattered near a magnet, though we may never make use of it, nor ever hope to become a Faraday. Is it due to the choice of our number system that there are some sets of integral lengths which form a right-angled triangle? Perhaps the mathematicians or the primary school teachers know an answer, but I do not, and I still wonder. Might there be another number system, perhaps there is, in which the great universal constants π, e, h, would be rational and even integral numbers? Or is it just that, as Sir James Jeans says, "There is a certain loose-jointedness in the universe"? To follow that thought into the realms of philosophy and morals and religion does not lower any standards, for there remain the dependability of the constants and the perfection of the eternal circle.

How often in the mathematics class do we burst the bounds of the material to get a glimpse of something simple, but beyond us. (Perhaps only too often!)

Let us consider the building up through school life of the following simple and related facts, in which the child can tread in the steps of nations. What more did the Egyptians need to think about the 3-4-5 triangle, except that with its aid they could mark out their fields again? When children are practising measuring inches, here is something practical to be discovered. And when they measure with ropes in feet and in yards the same shape comes again. (The germ of the principle of similarity.) Do other lengths than 3-4-5 give a square corner? . . . doubling? . . . ten times as long? . . . Do lengths of 4-5-6? . . . or 7-8-9? No. But there *are* some others . . we must look out for them. Or the children can be put on the track of 5-12-13.

And another discovery of 3-4-5, at some stage of school life. We may be teaching odd numbers, or square numbers, or merely practising addition, or looking for pattern, or leading directly to work on 3-4-5.

$$
\begin{array}{lll}
1 & = 1 & = 1^2 \\
1 + 3 & = 4 & = 2^2 \\
1 + 3 + 5 & = 9 & = 3^2 \\
1 + 3 + 5 + 7 & = 16 & = 4^2
\end{array}
$$

Arrange counters or dots to show the addition in patterns and perhaps a variety of patterns will be found. Which ones can be continued? Which is the simplest pattern? Will the pattern take a nine and be the same pattern? Sooner or later we shall arrive at squares. Can we arrange to use the last two squares to make the next? Notice the rhythm of the totals, even though the reason is obvious. This part of the work can be continued later, but we have at least found the 3-4-5 squares; by continuing the rows once more to add a nine $4^2 + 3^2 = 5^2$.

Let us approach 3-4-5 from another direction. In number work, set out all the numbers from 1 to 15 say, and under them, in blue, write their squares. (The teacher is there, after all, to some purpose: we are a sign-post to show the children paths which may lead somewhere, and it is a matter of experience to know how far to set them on the way, and this depends on the children with us at the time.) Is there any connection between the squares? Do any two squares together make a third square? Nearly always the 6-8-10 squares are noticed first. Are there any others? Wait, and they will be found. Is this true of any other three consecutive numbers? or of any other set of three? . . . only 5-12-13 among the numbers before us. Why should it be true of just these special ones?

More than that . . . why should it be that it is the same sets of numbers that show a special property in size and also in shape?

Can the squares of dots be connected with the sides of the right-angled triangles? The children will have ideas, and the art of teaching is to let them score the winning point. Then the two smaller squares could be made to fit over the big one (a first introduction to the later fact that $a^2 + b^2$ has no factors). There are commercial models sold in which the puzzle is to fit the pieces together. A flannel-graph is useful if a few of the class can come out and try. Text books show ways of cutting up the squares so that they will fit, and the important thing is for the children to perform the experiment individually. Sometimes it may be of value (perhaps always), to draw the triangle and the squares on its sides on graph paper also and actually count up the areas.

At some point we must move on to triangles with a square corner, but with the sides marked off at random; "tall, thin" triangles, or "wide, short" triangles. If these are the children's own descriptions of what they have drawn, is that not better than always being told what to draw? The new models also work and the graph paper work still gives the same general result. Where does the secret lie? If these triangles are found to be more easily drawn in a semi-circle, that "theorem" will become a reality. But now try triangles that do not have a right angle. The secret must lie with the right angle rather than with the lengths of sides. Is the element of wonder aroused, and does it lead to wondering "why"? The Greeks were faced with the same problm. How did their pure number link up with the facts of Egyptian geometry or surveying? Some of our children may previously have come to certain conclusions about areas, or we may pursue that now. They may be able, with Pythagoras, to follow a reasoned argument why the

geometrical results could not be otherwise than what their experiments have shown. They may perhaps skip that argument, with the promise that they will be able to reason it out later. Even if they cannot reproduce any argument, the reasoning part of their nature will have had some satisfaction. Reasoning power is not always well developed, perhaps through lack of anything to reason about or about which to wonder "why?". But it does not follow that the power to wonder why is lost, and there also remains the ability to see results found practically. After all, we do not know that the Egyptians did more than know the fact.

We still have to look out for other sets of numbers that would give the square corner. Let us continue the addition of odd numbers. The next line was

$$1+3+5+7+9=25=5^2$$

and this revealed (or we dug out of it) the three numbers of old acquaintance. When shall we have the sum of two squares again among the odd numbers? But how shall we find the total if we go on to 49, or, worse still, if we go on to 81? The class will be quite willing to add up, but is there no better, more exciting way? Many devices will be forthcoming (if the work is at the right stage).

Can we find a device where we do not need to know the row above, nor how many numbers in the row? If we can find the totals, a whole vista of Pythagorean numbers opens out before us. Observation should eventually disclose that if we stop at 99 the total will be 50^2. There is something satisfying in seeing a pattern and there is abundance of pattern here, and a sequence in all directions. To the children who discover the patterns . . . well, just watch their faces to know the satisfaction they have gained.

Then for those who can work in symbolism or even for those who appreciate that any letter can stand instead of any number, let us generalise the sum of odd numbers, and the reasoning part of us is assured that the sum could not be other than a square number.

For those who in school must be content to reason that results would be such and such if tried, is much satisfaction given, even to the reasoning part of their nature if the argument is largely repetition? If however there could be some practical result to reason about, the mind would be more active in its reasoning. Far better let all our children, whatever stream they are in, discover simple basic facts and experience the wonder that "it always seems to be so," and let those who can go on to reason that indeed "it always must be so."

"Let the children draw ANY triangle and measure the angles." "But they won't add up to 180° unless we give them the sides?" was the remark of a good student with G.C.E. mathematics. What sense of wonder had there ever been, if only certain triangles had this property?

Let the child-like attitude of wonder be developed, for here may lie a secret of humility. Let the inquisitive part of our nature be aroused, for this is somewhere near the roots of progress. Let us seek to bring out a sense of pattern and rhythm, for here chaos is replaced by order, and the mind knows that beyond itself there is something stable.

USING AIDS IN A GRAMMAR SCHOOL

R. M. FYFE

A critic tells me that aids are "for playing about" and that there is not time for this in the Grammar School, and then adds that, anyway, they *ought not* to be necessary.

It could also be said that pupils *ought not* to make mistakes. Serious mistakes occur because pupils do not understand. If we face this, we must accept the explanation that we failed to teach them, and the rationalisation that they were too stupid to learn leaves us with failure, just the same. That 30% or more of our pupils do fail to learn cannot be denied. Are we satisfied that nothing can be done about this?

In answer to the statement that there "is not time", my reply is that we have wasted 600 hours of the lives of all Grammar School pupils who fail to reach a satisfactory standard in their "O" level mathematics examination.

Nor am I interested in aids solely to help those who would otherwise fail: I am not convinced that we teach enough to the others, and particularly, I am not satisfied with the depth and consciousness of understanding of what mathematics is really about. Most pupils are convinced that it is a set of techniques for getting right answers to certain stereotyped questions.

Aids serve two purposes: to give experience of a situation that contains mathematics, and to stimulate the whole response of the pupil by bringing his motivating forces of feeling and interest into action. It is a commonplace of educational psychology that percept must come before concept. We also know that a word can do no more than communicate a meaning that is already understood. Insight into relationships is what we mean by mathematical understanding, and insight is the power which enables work to go forward. We create a mathematical situation in order to give opportunities for insight to occur.

Juggling about with figures and letters that stand for numbers, and with signs that stand for operations, is satisfactory when the meaning is understood. Or rather, we are able and willing to do this if we are expressing a movement (from out of our understanding) deliberately aimed from one point to another in an abstract argument. But a child who knows that $A^2 - B^2 = (A - B)(A + B)$ and also that $-(b - c) = -b + c$, and yet cannot factorise $(a - b)^2 - (b - c)^2$ instantly and correctly, reveals at once to us that insight into the mathematics of the situation has not yet taken place for him.

It must have happened to all of us that one of our pupils (because of an aid, or otherwise) has disconcerted us by saying: "Oh, *now* I see!" over some point we thought he understood long ago. It commonly happens, for example, when we are doing work involving three dimensions.

The purpose of this article is not to give "hints and tips" of partial solutions of teaching problems that known aids have made possible. Rather it is an invitation to those who think aids are a waste of time to answer two questions.

The first of these is: what aids do you use already (analogy to experience you

assume children have, drawing of static diagrams by the pupils, writing of figures and signs to stand for numbers and operations, and doing these things on the blackboard, etc.): why and how do you "draw the line", at what is useful and necessary, and what is a waste of time?

The second is: which of the mistakes that your pupils make reveal lack of insight, and how could you cater for the insight to take place sooner than it does?

Finally, I feel that I must state categorically that my experiments have proved conclusively that aids, wisely used, *save* time, and also make the children inclined to work much harder than they did when I relied exclusively on the blackboard and the written work they did in their exercise books.

ROUND TRIP

J. A. PRYDE

"If $f(z)$ is regular and continuous within and on a closed contour C except for a finite number of poles within C then integral $f(z)dz$ round C equals $2\pi i$ times the sum of the residues at the poles within C——."

A. L. CAUCHY, 1789-1857

Before I climbed aboard I ran my eye over the function. It was complicated, having arisen from a recalcitrant problem in diffraction theory. It looked regular enough, or at least meromorphic, but you never can tell with complex numbers. I've known the most demure and innocent-looking functions to behave outrageously on these trips, especially when they are a long way from the origin.

Our orders were to start along the real axis, so, when the *y*'s had made their farewells and been removed, we set out. For a time all went well and only the sound of a cosine gently oscillating broke the silence. We soon reached the unit circle, which was black with functions crawling slowly round it—a lot of mathematical work was in progress that day—and found a traffic jam had formed where it crossed the real axis. We passed that all right, apart from a few changes in sign—nothing serious—and ran along beside a deep ravine, where the plane had been cut to deal with a many-valued function which had been along this way a few days before.

Eventually we reached the turning where we stopped while the variables were changed. The Cartesians vanished and were replaced by thetas and rhos. I like the thetas, fat, jolly variables like Humpty Dumpty, with cummerbunds across their middles, but the rhos are thin, morose symbols. They have the aristocratic Greek temperament and are offended if you mistake them for common English *p*'s. After some trouble with the differential, we left the real axis at right angles and travelled along a circular contour to meet the *y*-axis.

The first ominous signs of disaster was a tragic scene a little way on. The function ahead of us had run slap bang into an essential singularity and had diverged instantly. It had been in charge of physicists of course, careless fellows who integrate quite recklessly, and bits of infinities of all orders were scattered all over the contour. We isolated the singularity and crawled round it in low gear, but it gave us all a bad moment. Then our troubles really began. I felt our function lurch and, to my horror, saw that the radius was tending to infinity! Tremendous

fights broke out all over the function. A power series was swelling rapidly and for a moment I thought all was over. But a negative exponential overpowered him with the terrific pressure from his minus sign and both disappeared as I watched. They're grand chaps, these exponentials, and very few functions can stand up to them in a divergence fight. The cosine however, was hysterical and shaking violently, threatening to upset our convergence, so I reached for the emergency control and applied Jordan's Lemma. After that, things quietened down and we reached the y-axis without further incident.

I looked at the integration meter. It read the same as when we left the x-axis! All that long haul round the circle had contributed nothing to the integral, not even an infinitesimal of the second order. We changed the variables again in disgust and set off for the origin and home.

The rest of the trip was uneventful—no singularities and only the tall graceful poles in the second quadrant to relieve the monotony of the flat landscape. Then across the unit circle again and so we tended gradually to zero.

I looked at our once proud and complex function. Where the intricate algebraic structure once stood, there remained only a few stolid constants and an integer or two—the residues. I collected them, multiplied them by $2\pi i$ and handed them in.

Another day's work was over.

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DIVIDE ET IMPERA

G. P. BEAUMONT

The usual technique of long division in algebra is an odd business. One fancies that the justification for the method is not clear to our pupils, though once having overcome the subtraction difficulty, they may use it well enough. Does this last remark constitute a defence of the standard method? Probably not, since our pupils are only too well accustomed to having drill handed out, and only complain when they can't do it. It is here suggested that the division process is best understood as an inverse operation. The following remarks are only intended to jog readers' memories and are not to be thought of as a detailed description.

The best check of the statement $\frac{17}{5}=3\frac{2}{5}$ is to find $5\times 3\frac{2}{5}$ or its equivalen $(5 \times 3) + 2$. The point about the remainder should be stressed. $17/5 = 3$ with remainder 2. What does 2 stand for? If this appears trivial, let the reader try asking his Third Form about the significance of the 3 in the statement: $(x^2 - 5x + 7)/(x - 1)$ has quotient $x - 4$ with remainder 3.

The nature of the result is best seen as:

$$\frac{x^2 - 5x + 7}{x - 1} \equiv (x-4)+\frac{3}{x-1}$$

Ah! the identity sign rears its ugly head. And why not? Our insistence on the sacred cow called "solving an equation" all too easily persuades our pupils that every algebraic situation must lead to a numerical value for x. It is fruitful to discuss propositions which may be: (i) true always; (ii) true for all but a

finite number of cases; and (iii) false for all but a finite number of cases. One of our own pupils pointed out that a statement which is true for almost all x is also true for particular values of x. (Which reminds us that division may be checked by means other than the Remainder Theorem, for what should we do on dividing by $x^2 - x + 8$?)

To return to our muttons, suppose

$$\frac{x^2 - 5x + 7}{x - 1} \equiv Ax + B + \frac{C}{x-1}$$

The form of the answer raises some interesting points. A Second Form only requires to know that there are no higher powers of x and that we can "stop" the division process when we like. The Sixth Form, on the other hand, might also show that $C/(x - 1)$ is the sum of an infinite series of terms like C_i/x^i which might have been included (though not without restricting the domain of x). Incidentally, why not divide out in ascending powers of x for a change?

The values of A, B and C may be readily found from the following steps:

$$\begin{aligned} x^2 - 5x + 7 &\equiv \left[Ax + B + \frac{C}{x-1}\right](x-1) \\ &\equiv Ax^2 + x(B-A) + (C-B) \end{aligned}$$

giving ample scope for revising taking out brackets and collecting terms. The "identification" of A, $(B - A)$ and $(C - B)$ lends some point to the mysterious examples we set about equating coefficients. Our readers will easily find the link between the above and the usual method for carrying out a division. It is not suggested that the inverse method is a complete substitute for the usual method, but rather that its use will strengthen the connections of division with other operations in algebra.

NEW TECHNIQUES IN MATHEMATICAL FILMS

Some of the work which we in the Film Unit have been doing recently has revealed new techniques which we hope will interest not only the few specialists who make such films but also those who look at them and who like to see new applications of mathematical ideas.

The most original development has been the production of a stereo film in colour showing electromagnetic oscillations in a resonant cavity. This is a specialised topic which does not concern many teachers, but it is a subject demanding stereo treatment because the oscillations can scarcely be illustrated in any other way. The lines of force are twisted curves which not only move but appear and disappear in a way which cannot be imitated by a mechanical model. We hope to be able to demonstrate this film at some of the Association's meetings, but not all projectors are suitable, also a metallised screen is necessary, and, of course, the audience must have polarised spectacles. It has been shown successfully in an auditorium seating over one hundred people at a demonstration given to the Scientific Film Association, and recently it received its first performance abroad—on the other side of the Iron Curtain at the Zentralinstitut für Lehrmittel in East Berlin.

The film was made by preparing two sets of drawings, one for each eye,

employing conventional peg animation techniques. The central problem was the small parallaxes (differences between the two images) which stereo drawing always entails. Small inaccuracies in still drawings can easily be spotted if one looks closely, and it was possible that errors of this kind would be fatal in animation, where they might have caused a most disturbing flicker in depth. Fortunately our experiments have shown that the eyes tend to average out these small random errors and so the production of three-dimensional diagrams by this method is a feasible proposition. In fact the eyes seem to be much more susceptible to transverse flicker than to slight variations in parallax. This is easily tested by shutting one eye and looking only at one animation sequence at a time. The pair of sequences seen together appears to run more smoothly than either seen separately.

The film *Resonant Cavities* is in colour and this in itself is a new development, as we have no coloured mathematical films in general use in this country, and we do not know of any diagram work in which colour plays an essential part anywhere else. Good topics for stereo film are probably rare; but there is no doubt that the medium is fascinating and can give quite new sensations of space.

As we reported in an earlier issue of this bulletin a further grant from the Nuffield Foundation has enabled us to make some copies of this film available for experimental showings. We are most grateful for this.

The theory of stereo projection is an immediate application of projective geometry. Students have informed me gravely that projective geometry is a creation of pure reason having no application whatever—a sad comment on a branch of mathematics whose origins were as practical as any. The traditional theory of stereoscopy is easily worked out and the distortions (or rather the geometrical transformations) which are to be expected when parameters in the system are varied make a good problem at scholarship level. However these traditional theories cannot account for certain phenomena which are observed in the laboratory, and Luneburg and others have developed a highly complicated explanation based on hyperbolic geometry. Stereo film may therefore turn out to be a wonderful method of teaching non-Euclidean geometry; but this is a long way ahead.

Another flat film is ready to go under the camera—a film on four-point systems of conics. Drawing this has led to the devising of some interesting drawing techniques. A colleague, Mr. P. J. Wakely, has shown me how projective radial and linear scales can be used in projective constructions to do the work of protractor and ruler in Euclidean geometry. These methods could be applied in the classroom and would supply a variety of drawing exercises. The essential idea is to trace a range or pencil occurring in a figure on to transparent material, and then to transfer it to other positions in the diagram. A projectivity is determined by three points, so when three points on the scale are aligned correctly the rest are in position automatically. These methods enable complicated projective constructions to be done quite rapidly.

Our only regret in the Film Unit is that we have not done more to produce elementary mathematical films. But the amount which we can do is limited by the number of pairs of hands working. We have the facilities for making films for the teaching of mathematics at all levels and we have some ideas; we lack only people with the patience and determination to carry them through to the end. If we want these films we must make them ourselves. All we need is volunteers. Who will come forward?

T. J. FLETCHER.

TEACHERS' VIEWS OF THE CUISENAIRE METHOD

Messrs. D. Karatzinas and T. Renshaw of the University of Edinburgh Department of Education have published recently in the *Scottish Educational Journal* (September 19th, 26th, October 3rd) three short articles summarising the results of an enquiry designed to obtain the views of practising teachers on the usefulness of the Cuisenaire material in the teaching of arithmetic. Mr. Renshaw and the Editor of the *Journal* have kindly given permission for the extraction of some shortened replies from the articles.

The first two parts of the research consisted of questionnaires from forty teachers whose replies were given at the beginning of the Spring Term, 1958, after they had used the coloured rods for just over one term in Edinburgh Corporation schools, class Primary 1. Sample replies are as follows:

	Yes	*No*	*Unsure*	*No reply or other answer*
(a) Do you consider that by using the Cuisenaire material you have achieved better results (so far) than you might have achieved in the same time without the material?	27	4	8	1
(b) Do you think that with the aid of the material children tend to be less readily frustrated than formerly?	34	0	3	3
(d) Looking ahead, would you say that for a substantial number of pupils:				
(i) skill in addition may be more quickly and easily developed using the material?	37	0	3	0
(ii) skill in subtraction?	30	1	6	3
(iii) skill in multiplication?	26	0	4	10
(e) Do you think that difficulty may be experienced in weaning children away from the material?	5	31	4	0
(h) If time can be saved by using Cuisenaire material, do you think that this time might best be spent in seeking to raise standards of achievement in reading and composition?	27	8	1	4

The second questionnaire asked for more detailed replies from the teachers with more particular reference to classroom organisation and procedure. Some teachers thought it wise to give ten minutes of "free-play exploration" at the beginning of every lesson; others considered that this would be better at the end.

Most of them preferred the pupils to work in groups, although opinion varied on how the groups should be constituted. Practically no difference had been observed between boys and girls either in rate of learning or appeal of the material. Many teachers found that the checking of work was easier and quicker, that time was saved in laying out rods compared with the usual use of counters. At the time of the enquiry counters had largely been discarded as of less use.

Others points included:

"The children never seem to tire of handling the material and the teacher enjoys teaching with it."

"Although it would seem that the Cuisenaire method would produce results quickly, progress is indeed slow at the beginning, but once the principles are grasped there is little difficulty."

". . . the gap between the capacity for learning of the brightest and dullest child has also been brought forcibly to my notice."

"A good many dull children do not follow and cannot remember the colour names in the first instance."

Finally tests were given to a class of forty boys in one school after eighteen months' experience with the material (the experimental group) and to a similar class of 14 girls and 24 boys in another school taught without the material (the control group). The tests used were:

Moray House Picture Test II (Quotient)
Thurstone's Test of the "Primary Mental Abilities"
Schonell's Diagnostic Arithmetic Test.
A specially designed Fraction Test.

"On all the tests taken by both groups the mean scores of the two groups are exceedingly close. None of the differences is statistically significant at the 5% level.

"As regards the standard deviations, only the difference on section six (Addition) of the Schonell test merits attention. Here the scores of the experimental group were more closely grouped at the top end of the scale than were those of the control group.

"In summing up the results of the experiment one can point to the earlier start made by the experimental group in learning to multiply and divide and to work out sums involving fractions, a start which was well consolidated during the period of the investigation as may be judged from the test results. One can also point to the fact that at the end of the period of investigation the experimental group had gained a no less effective mastery than the control group over the processes of addition and subtraction. It remains to be seen whether and to what extent the apparently greater progress made by the experimental group relative to the control group is maintained as the children move on through the primary school. An answer to these questions must be our next objective."

We shall look forward to the results of Mr. Renshaw's continuing investigation.

JOHN V. TRIVETT.

HOW CAN A GROUP OF TEACHERS HELP EACH OTHER?

It was in November, 1957, that Mrs. R. M. Fyfe and her co-workers founded the Middlesex Branch of the Association. Quite naturally, the first two meetings were held at Greenford Grammar School, where she is head of the mathematics department.

At these meetings, new members were rapidly introduced to the many kinds of teaching aids now available: coloured rods, geo-boards, static and working models film-strips and films; and various models were displayed. In addition, discussions were held on the best methods of dealing with troublesome topics, such as the rule of signs in algebra, and members who had found good ways of teaching these topics explained them to the rest of the group.

Our third meeting was held at Coston's Girls' School. Mrs. Fyfe gave a demonstration lesson using the Cuisenaire rods, and Mr. R. H. Fielding outlined for us the mathematics programme at Mountgrace Comprehensive School, Potters Bar. Some particularly effective models, made by students of King's College, were on view.

It was to Mountgrace School, where he is head of the mathematics department, that Mr. Fielding welcomed the branch for its fourth meeting on October 4th, 1958. Many of us had thought for some time that the usual static diagrams were insufficient to give our pupils a clear idea of geometrical relations, but short of going to the trouble and expense of making a film, we had found no way of remedying the insufficiency. By using an ingenious paper strip technique, Mrs. Fyfe showed how it could be done without any special apparatus whatsoever. She applied her method to the kite and to the system of two circles with their common tangents and common chord. The results were both elegant and enlightening.

Mr. Fielding then gave a demonstration lesson on the angle bisectors of a triangle, making use of the two films of Nicolet on this topic. Later he spoke to us about the various visual aids that he uses, and showed us some that are still in the experimental stage.

By then members had seen all the new teaching aids in action, and our chairman, Mr. D. H. Wheeler, decided that the time had come to put to the meeting the following questions:

(a) Why are so many of us here?

(b) Do we learn anything at the meetings that we could not find out just by reading *Mathematics Teaching*?

(c) Is there anything of value that we can give to our colleagues that we could not give to an even wider public by writing articles?

(d) Is there anything of value that a group of teachers can do as an organised group that individual teachers could not do separately?

In order to discuss these questions, those present divided into four groups of more manageable size. My impressions of what happened are, of course, limited to the particular group in which I was. Our group had no doubts as to why we met. It was in the expectation of both giving and receiving. We were eager to pass on to our colleagues the methods that we had, in the classroom, found to be effective in dealing with difficulties, and in return we hoped to receive guidance ourselves.

But what sort of guidance were we looking for? Guidance in dealing with difficult topics, certainly; but it soon became apparent that we were looking for something more. Our basic problem, the one we really cared about, was: "What mathematics should we teach our pupils, and why?" For, during our discussion, we saw more and more plainly that only when we were clear in our minds about our aims would we be able to decide on the best means of attaining them.

It was pointed out that the grammar schools had formulated reasonable aims and devised a method of teaching that, in skilled hands, gave good results with boys and girls in the top 15% of the general intelligence range, but that there was still no general agreement as to suitable mathematical fare for that large part of the population whose intelligence is near average. We therefore came to the conclusion that the first thing that should be done by the Middlesex branch as an organised group was to make an investigation into the content of the Secondary Modern mathematics curriculum, and that a similar investigation of the work of grammar schools should follow.

This did not mean that we had lost interest in teaching aids or in ways of dealing with awkward topics. On the contrary, it simply meant that we had begun to put first things first.

When the other groups reported to the meeting, their conclusions were, it seemed to me, in general agreement with ours, though naturally their formulations differed.

I feel confident that if the branch decides, as an organised group, to investigate this basic problem, its conclusions will be of the greatest value for the future of mathematical education in this country.

C. A. WINYARD.

CONTINUITY IN MATHEMATICS TEACHING BETWEEN THE PRIMARY AND SECONDARY STAGES

An open meeting held at Dowsett High School for Girls, Boston Avenue, Southend-on-Sea, on Saturday, November 8th, 1958, attracted a mixed audience of teachers from South-East Essex and the Borough of Southend-on-Sea.

The meeting was opened on behalf of the A.T.A.M. by Mr. D. T. Moore, who presented the aims of the Association briefly before presenting the first member of the team of lecturers, Mrs. R. M. Fyfe. A demonstration lesson was given to 9-year-olds using Cuisenaire rods. This preceded a short exposition of the scope of the rods by Mrs. Fyfe which some members of the audience took to be a demonstration in sleight of hand. Mrs. Fyfe assured them however that this was not intended, and that the facility with which a teacher could handle them obviously influenced the amount of help to be got out of the rods as a teaching aid. Mrs. Fyfe in a supreme effort to display the full powers of the rods rather crowded her lesson with subject matter and at least six out of the twenty-four members of the class took little part in the middle part of the proceedings. Those who did seemed to grasp facts with greater facility than they might have done using normal methods with many varied aids rather than this unified system.

In the course of the lesson the classroom was able to become acquainted

with sequence and commutation of integers, with the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, and with the terms applied to the elements of solids and the ways of finding the surface area of parallelepipeds. Mrs. Fyfe thought that whenever a mathematical term was proper it should be used, but admitted to the class that she often used others.

A great deal of discussion was aroused by the lesson. Some speakers in questioning the value of the rods gave the impression that they would use them as yet another mathematical aid but not as the sole system of imparting the properties of numbers. Others had the impression that the teacher was to become an instrument in a mechanical process of imparting number properties. The majority seemed to feel that any way of getting through the fact finding stage pleasantly inevitably ended in the stumbling block of fact remembering which, the older members of the audience seemed to feel, was best done the old way. "Learning the tables so as to produce a spontaneous answer to eight sixes" was the view put forward by one speaker. Another said that to be able to produce a number fact spontaneously should be a matter of class pride.

In replying to these views another speaker said that the old methods did not appear conspicuously successful in giving a realistic value of number properties to the child and quoted an example which had arisen out of the local school leaving certificate to illustrate the point. He thought that the Cuisenaire system of rods should be given a chance to prove their worth appealing as they do to the tactile and visual senses.

A demonstration using "Geoboards" was given by Miss J. E. Marsh in which the growth of squares and the introduction to irrationals was shown.

Miss Y. B. Giussepi gave an interesting talk on *Beginning in the Secondary School*. She favoured a completely fresh subject to start suggesting that a test or revision led to a bored child with, in the case of the secondary modern school, the added injury of repeating topics with which the child had so recently been unsuccessful. Her method was drawing and finding areas of odd shapes using squared paper. She favoured curve drawing and stitching, magic squares and any other topics that would hold the interest of a child and get him successful at something mathematical. An introduction like this she thought could be made over half of the first term with profit and the ensuing subject matter of the secondary modern child could in the first year closely parallel the subject matter of the grammar school first year.

Unfortunately the discussion following was rather short and many vital points brought out had to be left without comment.

The lack of time at the conclusion of the meeting with a rather rapid decrease in the numbers present led to the future plans for meetings in the area being rather vaguely traced. Some people were found, however, willing to act as the nucleus of a working group, before the chairman, after thanking the Headmistress of Dowsett Girls' High School for her kind co-operation, closed the meeting.

P. QUIGLEY

WEST RIDING GROUP

Thirty-two members attended the autumn meeting of the West Riding Group, held in Nightingale Primary School, Doncaster. The day's business commenced with progress reports from the sub-groups: Mr. K. Atkin described the work of the Modern School Group with their area project, and Mr. I. H. Runcieman told of the experiments already being conducted by the Grammar School Group in introducing algebra using Cuisenaire rods.

For the main part of the morning, Mr. C. Hope of Worcester Training College addressed the meeting on *The Third Year in the Training College*. He stressed the problems arising from the varying attainments of the students upon entering college and of the negative attitude towards mathematics which widely prevails. He believed that the task of the Training College was primarily to change this attitude and secondly, to increase mathematical ability and knowledge of the techniques of modern mathematics.

In the afternoon, Mr. Hope gave an entertaining, yet highly stimulating talk on graphical work. The concept of functionality is of the utmost importance in mathematics, but it was, Mr. Hope maintained, largely neglected in schools. He indicated how the simplest concept could be introduced in the primary school as a simple graph and developed to the senior stage in all secondary schools. He insisted that mechanical arithmetic should not be an end in itself but should yield data to be used in constructing a graph, which could clearly bring out the underlying functional relationship. Mr. Hope showed many models and novel examples of graphical work and the meeting was clearly impressed by the possibilities of the approach. The next meeting of the West Riding Group will be held on January 31st, 1959, when the teaching of area and the approach to geometry will be discussed. Interested members not already in the group can obtain full details from the Honorary Secretary, Mr. R. W. Shaw, 23 Pipering Lane, Scawthorpe, Doncaster.

R. W. S.

MEETING IN MANCHESTER

A one day course for all teachers of mathematics entitled *Secondary School Mathematics—Content and Method* will be held at Manchester University School of Education, Dover Street, Manchester, on Saturday, May 9th, 1959, at 10.15 a.m. There will be lectures, demonstration lessons and an exhibition of mathematical models and teaching aids. Speakers will include Mr. R. H. Cripwell, B.A., Didsbury Training College; Mr. C. Hope, B.Sc., City of Worcester Training College; Mr. R. H. Collins, B.Sc., Chairman of the Association; and Mr. C. Birtwistle, M.Sc., Nelson Secondary Technical School The full programme and other details are available from Mr. Birtwistle at 1 Meredith Street, Nelson, Lancashire.

SUBSCRIPTIONS

Subscriptions for 1959 should be sent to the Treasurer, 8 Glencairn Crescent, Edinburgh 12, as soon as possible. The annual subscription is 10s.

EARTH AND SKY

Professor S. Toulmin and Miss G. J. Goodfield of the Leeds University Department of Philosophy have recently completed a film entitled *Earth and Sky*. It is part of a programme to encourage interest in the history of scientific ideas, and it could profitably be shown to all members of the Sixth Form. The film shows how man's thoughts about the heavens in Babylonia and Greece changed and developed until in the seventeenth century the Copernican picture of the solar system replaced the Ptolemaic.

Some scenes in the film were shot in Greece and Iraq, and the film was made possible by the assistance which the producers received from a wide variety of helpers. Mathematicians will be particularly interested in the illustrations of the relative motions of the Earth and the planets which are given by machines specially made by the University Department of Mechanical Engineering.

We hear much about the need for understanding between followers of the Arts and the Sciences. This film, with the story it has to tell and with the values to which it bears witness, makes a practical contribution.

The film is 16mm. in colour, with sound, lasting 45 minutes. Provision is also made for running it in two halves. Further particulars may be obtained from Miss G. J. Goodfield, Department of Philosophy, The University, Leeds, 2.

A NEW TEACHING AID

A new aid to the teaching of mathematics has recently been marketed in this country, and a meeting of the Mathematics Club of the London Institute of Education was held on October 21st, 1958, at which Dr. Margaret Lowenfeld, the inventor of the material, demonstrated it and explained its purpose. Dr. Lowenfeld, who is Physician-in-charge at the Institute of Child Psychology, London, said that she hopes the aid will be used by teachers in the schools, although it was first conceived in the light of her experiences with maladjusted and misfit children at her clinic.

The aid consists of two sets of wooden blocks: "Poleidoblocs G" and "Poleidoblocs A". Poleidoblocs G are coloured and contain 54 pieces: 1in., 1½in. and 2in. cubes, divisions of the largest cubes into four congruent pieces in three different ways (triangular prisms and cuboids), flat rectangular blocks, circular cylinders, cones and square pyramids dimensionally related. These bricks may be used in free or guided constructions and the mathematical relationships become familiar, first in a tactile and then an intellectuel sense.

Poleidoblocs A are natural coloured and consist of a variety of rods of 1in. square cross-section, lengths from 1in. to 5in., cuboids obtained by dividing the rods in half, and triangular prisms ½in. thick. With these bricks, number concepts and properties can be developed by considerations of lengths and areas, and further geometrical propositions involving areas of squares, rectangles and right isosceles triangles dealt with at an early stage.

An assessment of the value of the blocks to teachers is difficult to make yet, as not enough experiment has been done or experience obtained in the classroom. That being the only valid test of the efficacy of any teaching aid, it is

important that it should be forthcoming and it is to be hoped that some teachers, particularly in the primary schools, will try out the material and report on their results. It is certain that much useful work could be done with individual pupils or small groups in giving the children familiarity with the shapes employed and the relationships between them.

Dr. Lowenfeld has said that she aims to preserve and utilise the curiosity of children concerning physical objects, fire their imaginations, and direct this drive into the acquisition of basic mathematical experience. She believes that this material can assist the child who finds mathematics meaningless, and reveal latent scientific ability in those who might not otherwise show it. D. H. W.

CORRESPONDENCE

Sir,

THE AVAILABILITY OF TEACHING AIDS

It is acknowledged that teachers of mathematics do not necessarily possess any ability with tools. This fact, coupled with the exorbitant prices of the narrow range of models available from existing sources, is, I am sure, the cause of many sound ideas being abandoned. To remedy this I should be pleased to receive specifications from teachers and to discuss experimental apparatus with them.

I am willing to construct such apparatus and feel that, coupled with an initial quotation regarding cost, this might well provide a much needed service. Will all who are interested please write giving me, initially, a general idea of their requirements.

D. J. LUMBARD,

"Wenvisue", Chandag Road,
Keynsham, Nr. Bristol.

Head of Mathematics Department,
Brislington County Secondary School,
Bristol, 4.

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Dear Sir,

In the November issue of your journal Mr. Macfarlane Smith of Newcastle advocates a Colour-Aid Number System which differs both from "Numbers in Colour" and from "Children discover Arithmetic". My own view is that all these "octave" systems are less helpful than a system which identifies three primary colours with the 4-2-1 binary digits respectively, with GREY for zero and a black "S" line superimposed for an inferred "−8"; the colours for denoting 3, 6 and 7 would then be mixed from the three primary colours, such as "orange" for 3, "green" for 6 and "brown" or "white" for 7.

Apart from colour there is much that we could do to simplify the teaching of arithmetic; we use the noughts-complements binary system for instructing inanimate computers, possessed of some 40 digits, but the system could easily be adapted to eight-fingered infants. In excluding thumbs from the count of fingers I have in mind the phenomenal powers of mental computing possessed by several of the children with super-numerary fingers—like the man of Gath. There may well be an explanation in terms of their possession of ten fingers for denary-scale computing—an advantage which would be enjoyed by normal infants only if the scale were changed from ten to eight—or perhaps to the pivoted-sixteen scale which I shall call *"Syncop-Eight" Diadic Counting*:

Negative numbers from 0		Positive numbers from 0
Oral S-N-T pattern	0	Oral S-N-T pattern
	NONN	
suss		nint
sesn		netn
sins		nutt
SONN	4	TONN
nuss		tint
nesn		tetn
nins		tutt
ATE	8	NINTYATE
sussytutt		nintynins
sussytetn		nintynesn
sussytint		nintynuss
SUSSYTONN	12	NINTYSONN
sussynutt		nintysins
sussynetn		nintysesn
sussynint		nintysuss
SUSSY	16	NINTY
sussysuss		nintynint
sussysesn		nintynetn

and so on.

With this "noughts complements" compounded-diadic system it is easy enough for the child to learn by heart two key patterns from which other products are simply obtained by deducting 1, 4, 9 . . . or 2, 6, 12 . . . in their SYNCOP-EIGHT form. These two key patterns are the n-squared and the n(n + 1) series—the series of squares and near-squares. The natural sequence of squares generates an expanded

series consisting of every fourth member of the natural series. Similarly, the natural series of near-squares generates an expanded series of squares by adding TONN (4 in denary) after shifting the digits for multiplication by 16. The pattern could easily be continued up to 1,600 and far beyond—without recourse to paper and pencil—but it is sufficient to give the first two "verses" of four "lines" each with four squares of the natural series.

1. nint tonn nintynins NINTY (1 to 16)
netnynins netnytonn nuttynint TONNY (25 to 64)
tintynint tetnytonn nint-atynins NINT-NINSY etc.
nint--nussynins nint-sonnytonn nint-nesnynint NINT-NONNY.

2. nint-netnynint nint-tonnytonn nint-nuttynins NETN-NINSY
netn-sonnynins netn-sesnytonn netn-nintynint NETN-TONNY
netn-tuttynint nutt-nesnytonn nutt-sesnynins NUTT-NINTY
nutt-tintynins tonn-atytonn tonn-sonnynint TONN-NONNY.

with verse 3 beginning "tonn-tonnynint tint-atytonn . . ." With the aid of either a human teacher or, preferably, an electronic trainer such as Mr. Gordon Pask's SAKI,it would not be long before even adults became familiar with the SYNCOP-EIGHT DIADIC computing, and even preferred it to denary just as we to-day prefer Arabic-Hindu computing to the MDCX work of Sir Thomas Gresham's Journals of AD 1546. Gresham was educated at Mercers' College and a German "Rechnungsbuch" of his boyhood is that by Jakob Koebel which was published at different times from 1514 up to 1537; even in the last edition the author has to apologise for the difficulties of the newfangled 123 system, giving this as his reason for beginning the book with MDCX instruction. His error on such an important matter is a warning to us to-day not to be complacent in accepting the second-best 123 system when our computers and electronic aids use a much more logical "diadic" system.

Yours faithfully,

D. S. BLACKLOCK.

Mr. Blacklock's letter raises many points of interest, and our present number habits should not blind us to the possible advantages of adopting a totally different system. We see the advantages of using bars in logarithms, but do not use subtraction in our number names at all. Nevertheless subtractive features appear in Russian and Hindustani numbers, and are systematically developed in the Yoruba language of Nigeria. The names in the system which Mr. Blacklock has invented have the ring of a "nonsense Jingle" about them, but precisely the same objection could raised (by those accustomed to British nomenclature) to the highly regular pattern of number names adopted in modern Finland.

But new systems have a great enemy in that highly conservative machine—the printing press. We are unable to print the special system of pictograms which Mr. Blacklock uses; just as we most regretfully could not publish an article on a system of automatic diadic numeration which has been invented by the world-famous cosmologist, Lemaitre.

—EDITOR.

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A little experiment will show the best kind of wire to use. Fabric covered wire is good as it fits snugly into the tube and does not wobble about; but lead wire, if you can get it, is perhaps the best of all.

BOOK REVIEWS

Eight-a-Day, Nine-a-Day, Ten-a-Day—Mental Arithmetic

A. L. Griffiths. Oliver & Boyd. 2/9d. each.

These books are designed to provide daily practice in Mental Arithmetic for the "A" pupils in the last three years in Primary School. Average pupils move back a book so that Ten-a-Day is said to be suitable for First year pupils in Secondary Schools. There is a steady progression in the difficulty of examples with regular introduction of new rules and ideas. In the earlier tests the questions are numerical examples on the four rules with a gradual introduction of verbal problems. As mechanical drill the matter is sound though unoriginal in form and at times quite unreal. What is the justification for including—1 week 2 days ÷ 3 – ?

There seems to be some contradiction in the introduction for the author stresses the value of Mental Arithmetic to provide a brisk test but also emphasises that the pupils should be provided with a copy each and work at their own individual speeds. The layout of the books is rather unattractive if they are to be used in this way.

D.T.M.

Final Check in Arithmetic

A. L. Griffiths. Oliver & Boyd, 2/9d.

This book is intended to provide diagnosis and revision, together with final tests in preparation for the 11-plus. The general approach leans toward the Moray House style of test and covers the ground thoroughly. It is not a book designed to attract a child or to interest him in any mathematical ideas. It has no significance apart from the 11-plus selection tests. It should prove useful to those pupils who need a "little extra" to surmount that hurdle and would probably best be recommended to parents anxious to cram their children into a Grammar School.

D.T.M.

An Introduction to Co-ordinate Geometry

A. Barton. University of London Press. pp. 303. 21s.

This is an excellent book. Mr. Barton has not been content with a rehash of existing works on the subject; clearly much thought has gone into its preparation. His approach is original and his treatment thorough. As one reads the book the feeling is not one of learning series of facts about different curves and of tricks for solving particular problems, but rather of understanding the principles and methods of the subject. Many curve properties, in fact, are left for the student to prove for himself, an indication of the method being given.

The book commences with a consideration of curve-tracing and elementary analytical properties of simple curves. The conic sections are introduced in a novel and interesting way for a co-ordinate geometry book—as sections of a cone! Freedom equations are then studied, determinants being used where applicable, and a thorough treatment of the general conic follows. There is a useful chapter on loci and envelopes, and the book concludes with two interesting chapters, "Co-ordinate Systems" and "Some Interesting Curves", which give a glimpse of what lies ahead in the subject as well as giving a breadth to what has been done already.

The examples worked in the text are adequate and consider the relative merits of different methods of solution, where applicable. The exercises set for solution include some interesting practical application (e.g. mechanical methods of drawing

curves) and are clearly intended to give an appreciation and understanding of the subject rather than be just problems for solution.

One can make criticisms, of course; for example, I would prefer to see the section on common points of two conics brought forward in the text, but a full table of contents and an index facilitate changes in order of topics during actual teaching. Some people, too, may complain of the "wordiness" of the text, but Mr. Barton has had in mind the Sixth Form student who has to work with only occasional help from the teacher. For such a student this book will provide all that is needed up to "A" and "S" level, while for the student who is fortunate enough to have a teacher constantly at his elbow, a browse through this book will give him added interest and wider outlook. The same might be said for the teacher who reads it, too!

A First Geometry

Part 3. D. N. Straker. Published by G. Harrap & Co. Ltd.

This book completes the "O" level course for the G.C.E. examinations in Geometry and Trigonometry, and the last chapter goes even further.

As for the former parts, the book is well produced, the thirty-three constructions and fifty theorems presented formally and clearly with good diagrams. Sincere efforts are made to introduce practical implications and the exercises especially include many good diagramatic examples.

But the underlying doubt remains: does another book like this really advance the problem of presenting geometry as vital, dynamic and irresistible or is it only of value to those pupils who already have an interest, albeit one imposed by the contingencies of examinations? Is there no place, for example, for the pupils' own discoveries when one line is translated across two fixed lines? From this kind of intuitive work could evolve the six theorems which appear in the section on similar triangles. If this had happened the pupils would have had a unified experience, rich in meaning, self-discovered, after which the formal, reasoned development could follow. But without this can we deny any pupil's right to ask, "Why?" and "Of what use?"

To summarise then, the three books in the series are good to handle, are handsome typographically and contain all the facts, theorems, constructions and exercises used in G.C.E. courses. The fundamental question which we need to ponder upon, however, is whether the children are helped to think and to act more mathematically as a result of the conventional approach. It is of little value asking the pupils this question as we cannot know whether any answer given is not second hand. The pupil may reply as he feels he is expected to, from memory or from politeness; probably the only way we can be certain of his knowing is if he discovers the realities *for himself*.

JOHN V. TRIVETT.

The Mathematical Ideas and Principles which underlie the Construction of Main Courses in Mathematics

A memorandum prepared by a committee of the Mathematics Section of the Association of Teachers in Colleges and Departments of Education. pp 12.

Mathematics as an instrument of liberal education—"A cultural course in which full emphasis is laid upon ideas, concepts and underlying principles in their historical setting, together with their practical consequences and applications in the external world." This is the keynote of the memorandum. A report of this length

can be little more than a statement of principles and it is inspiring to find how right the principles are. Ideas and education in a social context come first; and examinations come last, with constructive suggestions offered in the hope that some good may come out of a necessary evil.

It is always easier to express high ideals than to realise them in practice, and the middle sections of this pamphlet dealing with the material to be embodied in courses are the weakest. The traditional syllabus is there with a few cautious suggestions that some of the ideas of statistics, matrices and modern algebra might be introduced, and that more use should be made of mathematical machines of various kinds. Some mechanics should be introduced, but manipulation should be reduced to a minimum, a wider syllabus studied, and calculus given a central place.

These suggestions are all in the right direction, but do they go half far enough? Is not an extensive study of modern mathematics essential? And is it not time to regard mechanics as only a very small part of the applied mathematics which should be taught in schools? These two questions are not separate. Mathematical logic and modern abstract algebra can be applied mathematics (as some American textbooks show), and the school syllabus will be powerless to keep pace with contemporary life until schools and training colleges teach some of the algebra that men have devised in the last hundred years. This is often simpler than traditional school algebra, it is more fundamental, and it has wider applications. The old distinction between pure and applied mathematics is no longer valid; in an age of mechanical translation and logical computers all mathematics is applied mathematics.

Training Colleges and Departments of Education have recently been victims of much criticism. Certainly it is our opinion that these institutions should command more respect from the teachers who have been through them and from other University departments than they appear to do. Many rank and file teachers are hard critics of the ideas which operate in Training Colleges; but this report makes one think again. Can it be that the rank and file are wrong and the Training Colleges are right? In this report the aims are high, and amid a certain superfluity of subordinate clauses we find the right ideals. But are the practical suggestions vigorous and revolutionary enough to carry them out? T.J.F.

Calculus

D. R. Dickinson, Harrap. Vol. I. pp. 431, 17s. 6d.; Vol. II, pp. 228, 13s. 6d.

There are so many good books on the calculus already that one is tempted to ask, "Why another?". But the question is out of place; this book is fully justified and many schools will want to use it. Vol. I starts from the beginnings of the subject and covers the syllabuses of all examination boards at the Advanced level of the General Certificate of Education, together with some Scholarship level work. Vol. II completes the calculus required for University scholarship examinations and goes considerably further.

The book is well-written, with many examples and solutions at all stages, and it will be particularly useful in schools where the boys working for University scholarships have to do a lot of work on their own, reading for themselves. There is a judicious balance between rigour, on the one hand, and sympathy for the

student who is meeting the ideas for the first time on the other. Some difficulties are discussed more fully in appendices. It is good to see so many examples which look forward to applications in mathematical physics.

The author has sought to meet the requirements of many types of student in one book, and it is not altogether clear for whom the second volume is suited, starting as it does with function of several variables and including chapters on the complex variable (as far as contour integration and residues), Laplace transformation and Fourier series. These chapters are well beyond the requirements of University scholarships and many University students could use them; but the book does not go quite far enough to meet their needs completely. The second volume is much slimmer than the first; could we suggest adding chapters on conformal transformations and partial differential equations? If this were done the book could be recommended to many degree students (General, and even Special), especially to the less able.

There are two minor criticisms. The name "Lhopital" reads very strangely, and whilst the diagrams are clear the curves in them are often drawn irregularly and spoil the appearance of the book. But we certainly congratulate the publishers on the price. As value for money the book is hard to beat. T.J.F.

ANOTHER NEW PARTICLE

We welcome the first number of a new periodical. *Particle*—a quarterly by and for science students has been founded in America by Mr. Dunbar Aitkens, who recently spent some time in Britain with the United States armed forces and who will, no doubt, be known to some of our readers. This issue contains an article on the I.B.M. 701 computer, and articles on physics, chemistry and biology. The subtitle indicates the aims and scope of this new venture and we wish it every success. The annual subscription is $1.20 and single copies cost 30c., and the editorial offices are at 2531 Ridge Road, Berkeley 9, California.

It is always difficult to guess in which continent Professor W. W. Sawyer will turn up next! *The Mathematics Student Journal*, published by the National Council of Teachers of Mathematics, U.S.A., is out in an attractive new format and is now under Professor Sawyer's editorship. Eight pages, two colours, the promise that all issues will contain two pages that can be understood by people who only know arithmetic, sections of articles and puzzles of increasing difficulty up to pages devoted to student discoveries (or rediscoveries), and a department for future research workers is the recipe for a bright and breezy magazine of school mathematics full of the happy spirit of do-it-yourself that we associate with its editor. The address for subscriptions is M. H. Ahrendt, 1201 Sixteenth Street, N.W. Washington 6, D.C., U.S.A. Subscriptions are sold only for five copies or more computed at a single copy rate of 30c. per year, 20c. per semester.

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